Prototype 1 – Problem of *P&T* Co. (HL[1], §8.1, page. 305)

One of the main products of the *P&T* company is canned peas. The peas are prepared in three canneries (near: Bellingham, Washington, **C1**; Eugene, Oregon, **C2**; Albert Lea, Minnesota, **C3**), and then shipped by truck to four distributing warehouses in the western United States (Sacramento, California, **W1**; Salt Lake City, Utah, **W2**; Rapid City, South Dakota, **W3**; Albuquerque, New Mexico, **W4**). Because the shipping costs are a major expense, management is initiating a study to reduce them as much as possible. For the upcoming season, an estimate has been made of the output from each cannery and each warehouse has been allocated a certain amount from total supply of peas. This information (in units of truckloads), along with the shipping cost per truckload for each cannery-warehouse combination is given in the table below. Thus, there are a total of 300 truckloads to be shipped. The problem now is to determine which plan for assigning these shipments to the various cannery-warehouse combinations would minimize the total shipping cost.

	Shipping cost (*m.u.*) per truck load				
Warehouses / Canneries	**W1**	**W2**	**W3**	**W4**	Output
C1	464	513	654	867	75
C2	352	416	690	791	125
C3	995	682	388	685	100
Allocation	80	65	70	85	

Prototype 2 – Problem of *JOB SHOP COMPANY* (HL[1], §8.3, page 334)

The *JOB SHOP* company has purchased four new machines of different types. There are four available locations in the shop where a machine could be installed. Some of these locations are more desirable than others for particular machines because of their proximity to work centers that will have a heavy work flow to and from these machines. (There will be no work flow between the new machines.) Therefore, the objective is to assign the new machines to the available locations to minimize the total cost of materials handling. The estimated cost in dollars per hour of materials handling involving each of the machines is given in the table below for the respective locations. Location 2 is not considered suitable for machine 2, so no cost is given for this case.

	Cost ($) of materials handling			
Location / machine	**L1**	**L2**	**L3**	**L4**
M1	13	16	12	11
M2	15	–	13	20
M3	5	7	10	6
M4	12	20	15	13

a) Formulate and solve the problem by Solver.
b) Formulate and solve the problem by Solver, assuming that the company do not buy **M4.**

Exercises of Transportation and Assignment Problems

21. Solve problems **8.1-2.** and **8.1-7.** from HL[1] (pages. 348-349).

A) **8.1-2.** The Childfair Company has three plants producing child push chain that are to be shipped to four distribution centers. Plants 1, 2, and 3 produce 12, 17, and 11 shipments per month, respectively. Each distribution center needs to receive 10 shipments per month. The distance from each plant to the respective distributing centers is given below:

		distance			
		distribution center			
		1	2	3	4
plant	1	800 miles	1300 miles	400 miles	700 miles
	2	1100 miles	1400 miles	600 miles	1000 miles
	3	600 miles	1200 miles	800 miles	900 miles

The freight cost for each shipment is $100 plus 50 cents per mile. How much should be shipped from each plant to each of the distribution centers to minimize the total shipping cost?

(a) Formulate this problem as a transportation problem by constructing the appropriate parameter table.

(b) Draw the network representation of this problem.

(c) Obtain an optimal solution.

B) **8.1-7.** The Move-It Company has two plants producing forklift trucks that then are shipped to three distribution centers. The production costs are the same at the two plants, and the cost of shipping for each truck is shown for each combination of plant and distribution center:

		distribution center		
		1	2	3
plant	A	$800	$700	$400
	B	$600	$800	$500

A total of 60 forklift trucks are produced and shipped per week. Each plant can produce and ship any amount up to a maximum of 50 trucks per week, so there is considerable flexibility on how to divide the total production between the two plants so as to reduce shipping costs. However, each distribution center must receive exactly 20 trucks per week.

Management's objective is to determine how many forklift trucks should be produced at each plant, and then what the overall shipping pattern should be to minimize total shipping cost.

(a) Formulate this problem as a transportation problem by constructing the appropriate parameter table.

(b) Display the transportation problem on an Excel spreadsheet.

(c) Use the Excel Solver to obtain an optimal solution.

22. Three refineries (**R1**, **R2** and **R3**) with a daily production capacity of 25'000, 15'000 and 5'000 *ton.* of gas, respectively, supply three large distribution centers (**D1**, **D2** and **D3**) which needs are respectively 15'000, 10'000 and 20'000 *ton.*. The supply is done throughout a pipeline network in a price of 200 *m.u.* per *ton.*, per *km*.. Distances between refineries and distribution centers (in *km*) are given in the table below:

	D1	**D2**	**D3**
R1	*5*	*70*	*320*
R2	*75*	*15*	*220*
R3	*300*	*200*	*2*

a) Formulate the problem as a linear programming problem.

b) Find the optimal solution.

c) Solve the problem assuming that refinery **R2** stopped the production of gas.

d) Solve the problem considering that needs in distribution center **D3** are now 10'000 *ton.*.

e) Solve the problem considering that the production in refinery **R1** is equal to 20'000 *ton.*.

23. A factory has four machines and four tasks that could be performed by any of the machines. Each machine should perform a task from the beginning until the end. The time required by each machine (in hours) to complete each one of the tasks is given in the following table.

tasks / machines	**T1**	**T2**	**T3**	**T4**
M1	*14*	*5*	*8*	*7*
M2	*2*	*12*	*6*	*5*
M3	*7*	*8*	*3*	*9*
M4	*2*	*4*	*6*	*10*

Determine the solution that minimizes the time needed to perform the four tasks, assigning a task to each of the machines.

24. A department has opened three vacancies for translators:

Vacancy 1: Portuguese/French;

Vacancy 2: Portuguese/German;

Vacancy 3: Portuguese/Greek.

Four candidates applied and in the selection tests they achieved the following grades (in scale from a minimum of zero to a maximum of ten):

	Vacancy 1	**Vacancy 2**	**Vacancy 3**
Candidate 1	7	6	2
Candidate 2	8	8	4
Candidate 3	8	5	4
Candidate 4	9	7	6

Determine the assignment that provides the best service quality.

25. A company produces a product in two factories (**F1** and **F2**) and has three selling points (**S1**, **S2** and **S3**). The maximum production for the next period is 400 *ton.* and 800 *ton.* in factories **F1** and **F2**, respectively. The potential sales in the three selling points are 400 *ton.*, 500 *ton.* and 500 *ton.,* respectively. The transportation cost, in *hundreds of m.u.* per ton transported, between each factory and each selling point are in the following table:

	S1	**S2**	**S3**
F1	10	20	25
F2	25	15	20

The product is sold by 15, 18 and 20 *thousands of .m.u. per ton* in selling points **S1**, **S2** and **S3** respectively and the management of the company wants to maximize the total profit (revenue - cost). Determine the optimal solution.

26. Formulate the following examples adapted from HL[1]:

A) A *METRO WATER DISTRICT* (HL[1], page 316) is an agency that administers water distribution in a large geographic region, the main customers are four cities (Berdoo: **C1**; Los Devils: **C2**; Sam Go: **C3** and Hollyglass: **C4**) and the water supply is from three rivers (Colombo: **R1**; Sacron: **R2** and Calorie: **R3**).

It is possible to supply any of the cities with water from any of the rivers, except **C4**, that cannot be supplied by **R3**.

The costs (in *m.u.*) of sending one million *Kl* of water from river **Ri** to city **Cj**, are in the table below, as well as the availabilities and needs.

river \ city	**C1**	**C2**	**C3**	**C4**	availabilities (*millions of Kl*)
R1	16	13	22	17	50
R2	14	13	19	15	60
R3	19	20	23	—	50
Minimum needed (*millions of Kl*)	30	70	0	10	
Requested (*millions of Kl*)	50	70	30	∞	

Management wishes to allocate all available water from the three rivers to the four cities in such a way as to at least meet the essential needs, while minimizing the total cost.

B) The company *BETTER PRODUCTS* (HL[1], pág. 339) has decided to initiate the production of four new products (**P1**, **P2**, **P3** and **P4**), using three plants (**F1**, **F2** and **F3**) that currently have excess production capacity. The products require a comparable production effort per unit, so available production capacity of plants is measured by the number of units of any product that can be produced per day, as given in the table below. The bottom row gives the required production rate per day to meet projected sales. Each plant can produce any of these products, except that plant **F2** cannot produce product **P3**. However, the variable costs per unit of each product differ from plant to plant, as shown in the main body of the table:

	Product unit cost (*m.u.*)				
Plant (Factory) \ **Product**	**P1**	**P2**	**P3**	**P4**	Daily capacity available (*units*)
F1	41	27	28	24	75
F2	40	20	–	23	75
F3	37	30	27	21	45
Production rate per day (*units*)	20	50	30	50	

Management now needs to make a decision on how to split up the production of the products among plants. Two kinds of options are available.

i) Permit product splitting, where the same product is produced in more than one plant;

ii) Prohibit product splitting and each plant can only produce one product.

27. A couple wants to share some tasks in order that the total time spent is minimized, but both should do the same number of tasks. The average weekly time (in *minutes*) needed, for each one to do the tasks is the following:

	Shopping	Cooking	Dish washing	Laundry	House cleaning	Make bed
João	60	400	150	210	65	70
Ana	90	300	100	180	90	40

a) Determine the tasks that should be assigned to each one.

b) How long, per week, will be spent by each one in the tasks assigned?

28. A company has four vacancies: **V1**, **V2**, **V3** and **V4**. According to the psychologist, that vacancies should not be given to individuals with an I.Q. (intelligence quotient)**,** lower than, 150, 100, 80 and 75, respectively.

Five candidates apply: **C1**, **C2**, **C3**, **C4** and **C5** (to any of the four vacancies) and the IQ tests performed assign I.Q. scores of 190, 160, 145, 100 and 85, respectively.

The monthly wage asked by the candidates was 150, 80, 100, 100 and 70 *u.m.*.

What is the optimal assignment and respective cost, assuming that no more candidates applied and the first was immediately hired?

29. A company decided to produce three new products, **P1**, **P2** and **P3**. Currently, the company has five factories with capacity excess. The unit production cost (in.*m.u.*) of the first product is 12, 10, 13, 11 and 12 in factories **F1**, **F2**, **F3**, **F4** and **F5**, respectively. For the second product these costs (in.*m.u.*) are, 5, 4, 6, 3 and 4, respectively. The unit costs (in.*m.u.*) of the third product are 9, 7 and 9 in factories **F1**, **F2** and **F3**, factories **F4** and **F5** cannot produce it. The sales forecasts are 3000, 3000 and 2000 *units* of products **P1**, **P2** and **P3**. The factories **F1**, **F2**, **F3**, **F4** and **F5** have a capacity to produce 2500, 3000, 2000, 4000 and 5000 *units* of those products, respectively, and the product or products combination is not relevant.

Consider the following *output* from the *Solver*, displaying the optimal solution for the problem:

Target Cell (Min)

Cell	Name	Original Value	Final Value
$G$19	Total cost	0	56000

Adjustable Cells

Cell	Name	Original Value	Final Value
$C$14	F1 - P1	0	0
$D$14	F1 - P2	0	0
$E$14	F1 - P3	0	0
$C$15	F2 - P1	0	1000
$D$15	F2 - P2	0	0
$E$15	F2 - P3	0	2000
$C$16	F3 - P1	0	0
$D$16	F3 - P2	0	0
$E$16	F3 - P3	0	0
$C$17	F4 - P1	0	2000
$D$17	F4 - P2	0	2000
$E$17	F4 - P3	0	0
$C$18	F5 - P1	0	0
$D$18	F5 - P2	0	1000
$E$18	F5 - P3	0	0

a) Write the optimal solution and explain it in economic terms.

b) By technical and logistic reasons, the management decided that each factory either will not produce any product or will produce only one, and that each product can only be produced by one factory. Which should be the new production plan?

30. A company that sells cars is going to open two new shops (**NA**, **NB**) with space for 30 cars each one. For the moment, no cars are available at the factory, it was decided that the cars should be shipped from the four closest shops (**V1**, **V2**, **V3** and **V4**). Each one of the shops offered no more than 20 vehicles to be transferred. The link between **V1**-**NA**, is not available due to ongoing street repair.

Knowing that the new shops should receive the maximum number of cars and that the transferring unit costs (in *m.u.*) are in the table below,

	NA	NB
V1	-	170
V2	230	140
V3	170	130
V4	200	150

fill the sheet attached (appendix **B**) in order that the problem could be solved by *Solver/Excel* (write exactly what would be written in case a computer is available).

31. Consider the problem referring to the transportation of an item from three warehouses to three shops. The unit costs, supplies, demands and *Solver output* are:

	Shop 1	**Shop 2**	**Shop 3**	Supply
Warehouse 1	4	6	8	40
Warehouse 2	2	4	2	20
Warehouse 3	6	-	4	30
Demand	20	50	40	

Target Cell (Min)

Cell	Name	Original Value	Final Value
$G$16	Total cost	0	380

Adjustable Cells

Cell	Name	Original Value	Final Value
$C$13	Warehouse 1 - Shop 1	0	10
$D$13	Warehouse 1 - Shop 2	0	30
$E$13	Warehouse 1 - Shop 3	0	0
$C$14	Warehouse 2 - Shop 1	0	10
$D$14	Warehouse 2 - Shop 2	0	0
$E$14	Warehouse 2 - Shop 3	0	10
$C$15	Warehouse 3 - Shop 1	0	0
$D$15	Warehouse 3 - Shop 2	0	0
$E$15	Warehouse 3 - Shop 3	0	30

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$F$13	Warehouse 1	40	$F$13=$H$13	Not Binding	0
$F$14	Warehouse 2	20	$F$14=$H$14	Binding	0
$F$15	Warehouse 3	30	$F$15=$H$15	Binding	0
$C$16	Shop 1	20	$C$16<=$C$18	Binding	0
$D$16	Shop 2	30	$D$16<=$D$18	Not Binding	20
$E$16	Shop 3	40	$E$16<=$E$18	Binding	0
$D$15	Warehouse 3 - Shop 2	0	$D$15=0	Not Binding	0

a) Explain how the transportation should be done.

b) What changes should be introduced in the model defined in *Excel* and in the specifications file of *Solver* to ensure that **Shop 2** receives the quantity demanded and that **Warehouse 2** send, exactly 10 units to that shop.

32. The following table displays the data of a problem that arose in a company that produces a product in four factories, **F1**, **F2**, **F3** and **F4**, to be sold in four markets, **M1**, **M2**, **M3** and **M4**. Row **D** represents the demand to meet in each market (in *ton.*) and column **S** the maximum capacity of each factory (in *ton.*). The remaining values are the production and transportation costs of each ton of product (in *m.u.*).

	M1	**M2**	**M3**	**M4**	**S**
F1	5	8	4	7	23
F2	2	6	6	6	32
F3	3	7	5	7	38
F4	2	5	4	3	38
D	21	16	30	35	

The conclusions of a study dictated that each market should be supplied by only one factory, provided it has enough capacity. On the other hand, it was decided that all factories must operate. Formulate the problem defining variables constraints and the objective function to optimize.

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[1] Hillier, Lieberman, "Introduction to Operations Research", 9th edition, McGraw-Hill, 2010.